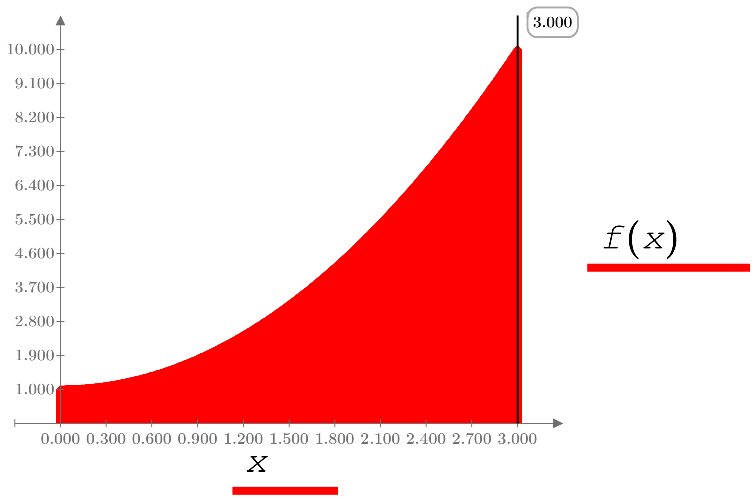
Numerical Methods

Consider the problem of determining the area under the curve $f(x) := x^2 + 1$, with x between $a := 0$ and $b := 3$. This is done by approximation method.



$N := 5$

$i := 1, 2 .. N$

$h(N) := \frac{b - a}{N}$

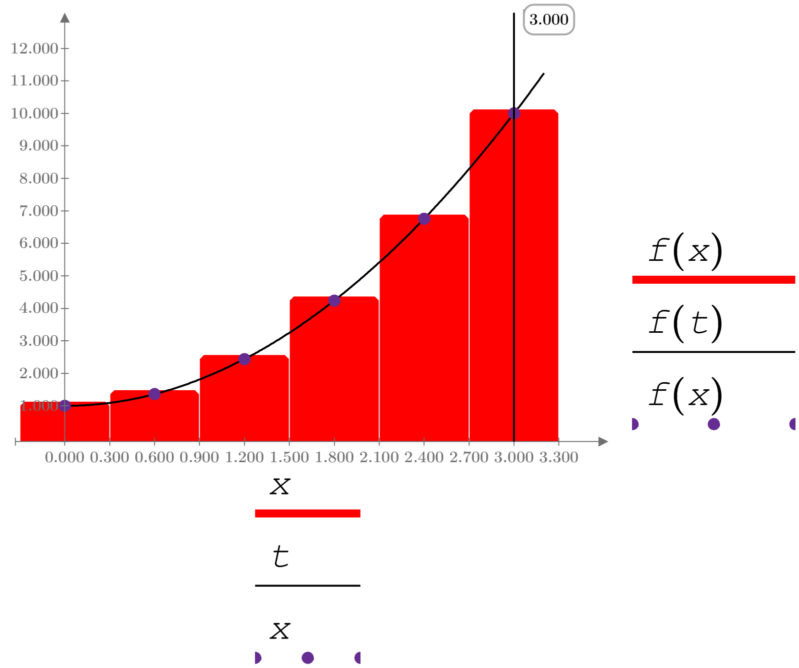
$x_0 := a \qquad x_i := i \cdot h(N)$

$$\sum_{i=0}^{N-1} h(N) \cdot f(x_i) = 9.480$$

$$\sum_{i=1}^{N} h(N) \cdot f(x_i) = 14.880$$

$$\int_a^b f(x)\, dx \rightarrow 12$$

$$\sum_{i=0}^{N-1} h(N) \cdot f\left(x_i + \frac{h(N)}{2}\right) = 11.910$$



$$R(N) := \sum_{i=0}^{N-1} h(N) \cdot f\left(\left(i + \frac{1}{2}\right) \cdot h(N)\right)$$

$R(N) = 11.910$

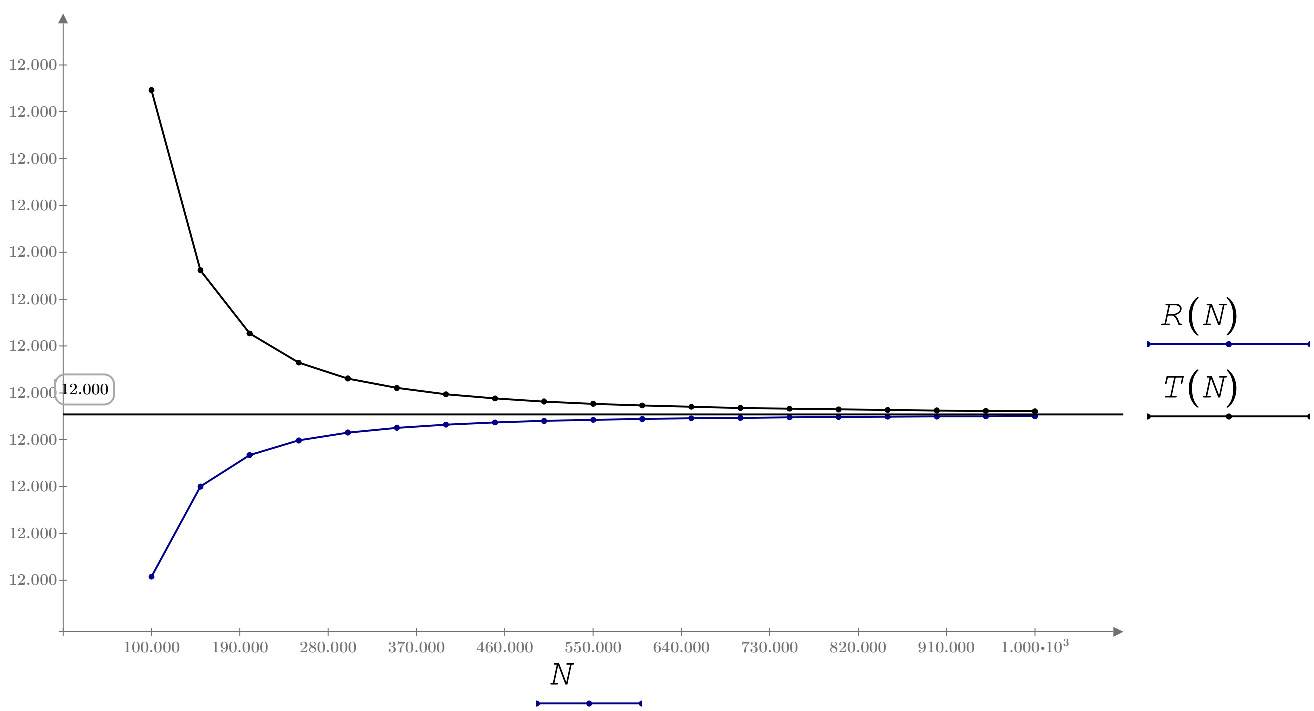
$R(100) = 12.000$

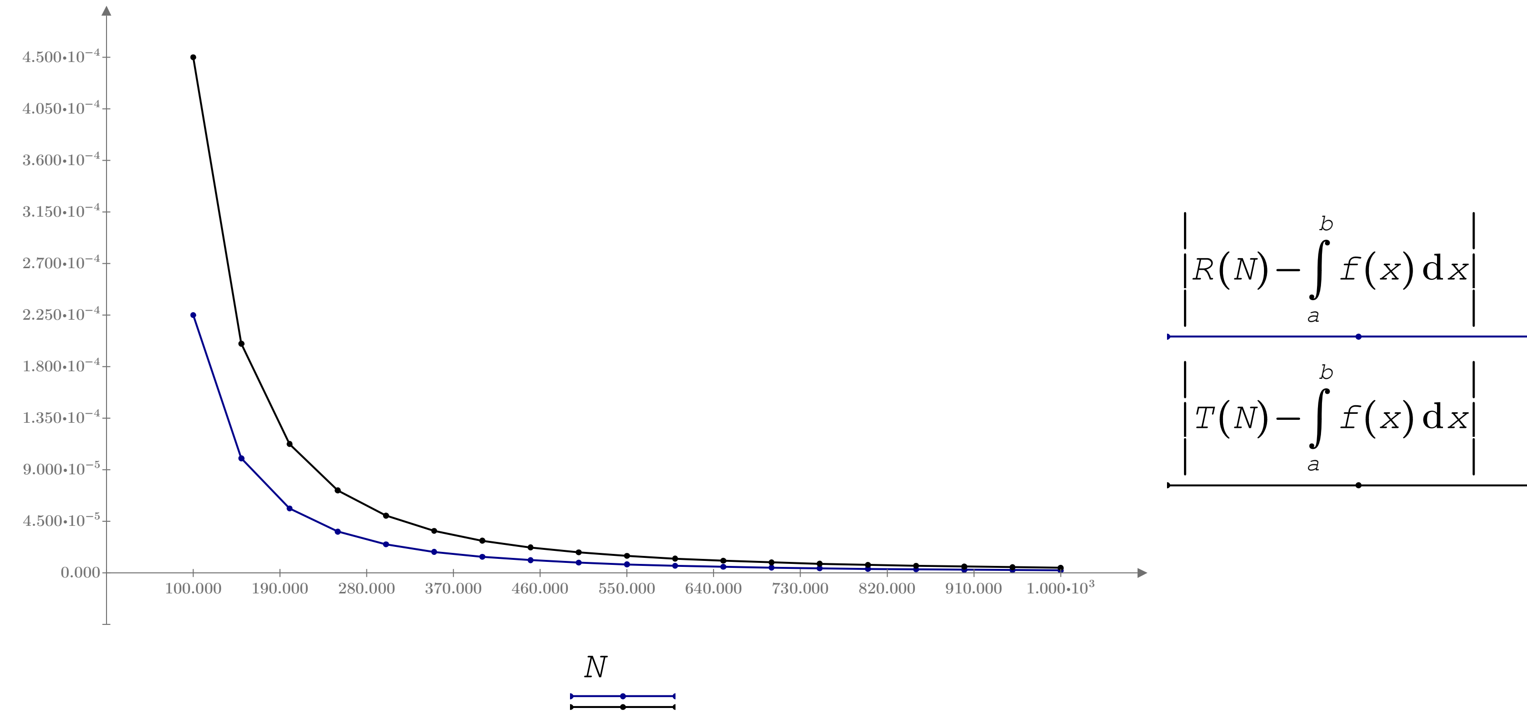
$$T(N) := \frac{f(a) + f(b)}{2} \cdot h(N) + h(N) \cdot \sum_{i=1}^{N-1} f(i \cdot h(N))$$

$T(N) = 12.180$

$N := 100, 150 .. 1000$

$T(100) = 12.000$





References:

1. **G.Simmons.** Calculus With Analitic Geometry (1996): 10.9.
2. **MIT** lection 03.19.2013.