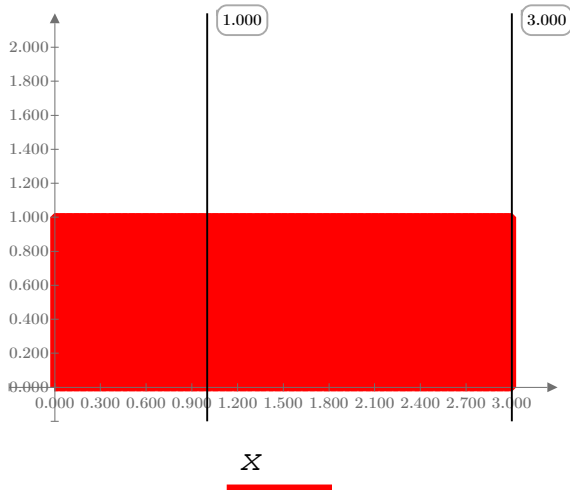


Definite Integrals

Example 1. Consider the rectangle, i.e. graph of $f(x) := 1$, with x between $a := 1$ and $b := 3$. The square of the rectangle is $S := f(a) \cdot (b - a) = 2.000$.



$$\int_a^b 1 \, dx = 2.000$$

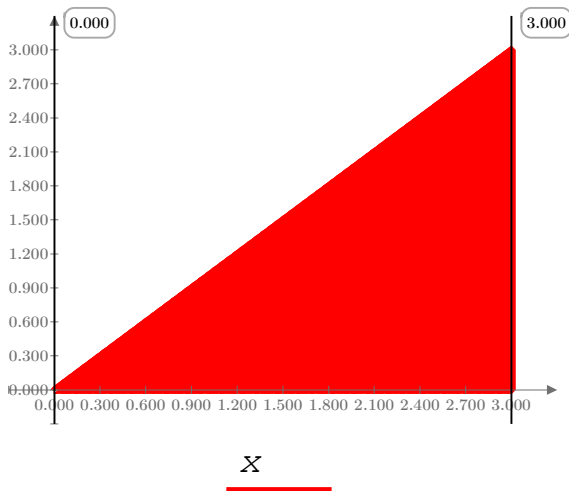
$$N := 5$$

$$h := \frac{b - a}{N}$$

$$\sum_{i=0}^{N-1} h \cdot 1 = 2.000$$

$$b - a = 2.000$$

Example 2. Consider the triangle, i.e. graph of $f(x) := x$, with x between $a := 0$ and $b := 3$. The square of the triangle is $S := \frac{f(b) \cdot (b - a)}{2} = 4.500$.



$$\int_a^b x \, dx = 4.500$$

$$\int_0^B x \, dx \rightarrow \frac{B^2}{2}$$

$$N := 20$$

$$i := 1, 2 \dots N$$

$$x_0 := 0 \quad x_i := i \cdot h(N)$$

$$h(N) := \frac{b-a}{N}$$

$$\sum_{i=0}^{N-1} h(N) \cdot f(x_i) = 4.275$$

$$\sum_{i=1}^N h(N) \cdot f(x_i) = 4.725$$

$$\sum_{i=0}^{N-1} h(N) \cdot f\left(x_i + \frac{h(N)}{2}\right) = 4.500$$

$$\lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} h(N) \cdot f(h(N) \cdot i) \rightarrow \frac{9}{2}$$

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N h(N) \cdot f(h(N) \cdot i) \rightarrow \frac{9}{2}$$

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N h(N) \cdot f\left(h(N) \cdot \left(i + \frac{1}{2}\right)\right) \rightarrow \frac{9}{2}$$

$$\int_a^b f(x) dx \rightarrow \frac{9}{2}$$

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \Delta x \cdot f(X_i) = \int_a^b f(x) dx$$

References:

1. **G.Simmons.** Calculus With Analytic Geometry (1996): 6.4, 6.5.
2. **MIT** lection 03.19.2013.